Longitudinal Beam-Dynamics Constraint on Accelerating Gradient

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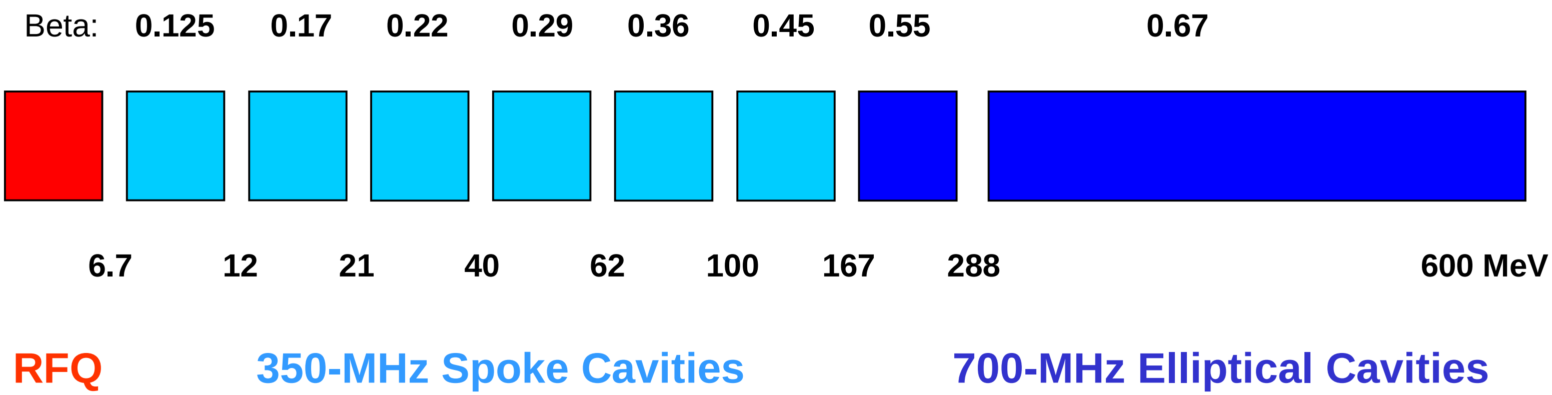
High-gradient 20-mA compact CW proton superconducting-linac design for testing of accelerator-driven nuclear waste-transmutation target concepts.



- Our design extended the superconducting linac to much lower velocities all the way down to the RFQ.
- 5-cell spoke cavities were used in first three spoke sections to increase the real-estate accelerating gradient at low beta; 7-cell cavities (spoke and elliptical) were used for entire remaining linac.
- **We found that using high accelerating gradients at low velocities produced the longitudinal envelope instability.**
- Design modifications to be described mitigated the effect and allowed higher gradients to be used.

Longitudinal envelope instability is excited if the accelerating gradient is too large.

- **Longitudinal rms beam size is resonantly driven** (**parametric resonance**) **to larger values by the periodic focusing from rf cavities** when focusing lattice period $L_{period}/\beta c$ is half the period of a mismatch oscillation.

- **Thus, resonance occurs when the longitudinal phase advance per focusing period of longitudinal mismatch oscillation equals 180°.**

Model for longitudinal rms envelope Z in periodic focusing lattice with period L.

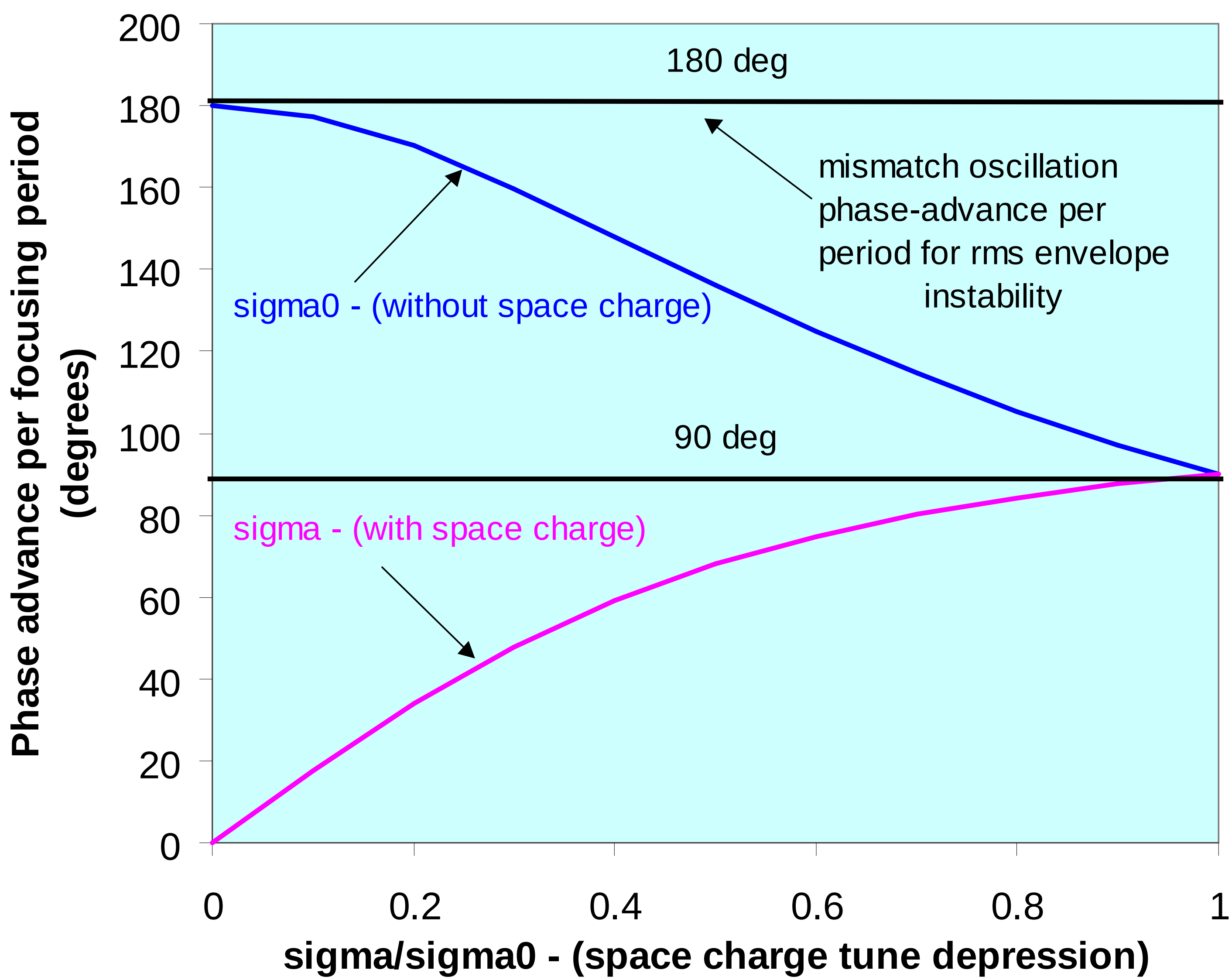
$$Z'' + \left(\frac{\sigma_0}{L}\right)^2 \left(1 + \delta \sin\frac{2\pi s}{L}\right) Z - \frac{\varepsilon^2}{Z^3} - K = 0, \quad longitudinal.$$

$$Let\ \ Z = Z_0 + z,\ \ for\ small\ mismatch\ z.$$

$$z'' + \left(\left(\frac{\sigma_0}{L}\right)^2 + \frac{3\varepsilon^2}{Z_0^4}\right) z + \left(\frac{\sigma_0}{L}\right)^2 \delta \sin\frac{2\pi s}{L}\, z = 0, \quad parametric\ resonance.$$

$$\text{Resonance}: \sigma_{mismatch} = 180^\circ,\ where \left(\frac{\sigma_{mismatch}}{L}\right)^2 = \left(\frac{\sigma_0}{L}\right)^2 + \frac{3\varepsilon^2}{Z_0^4}$$

Single Particle Tunes in Smooth Approximation Corresponding to Longitudinal Envelope Instability



We can control σ_0 with external focusing. Safe criterion to avoid envelope instability for all beam currents is $\sigma_0 < 90°$. This limits the average accelerating gradient $\langle E_0 T \rangle$.

$$\left(\frac{\sigma_0}{L}\right)^2 = \frac{2\pi q \langle E_0 T \rangle \sin(-\phi)}{mc^2 \gamma^3 \beta^3 \lambda} \le \left(\frac{\pi}{2L}\right)^2.$$

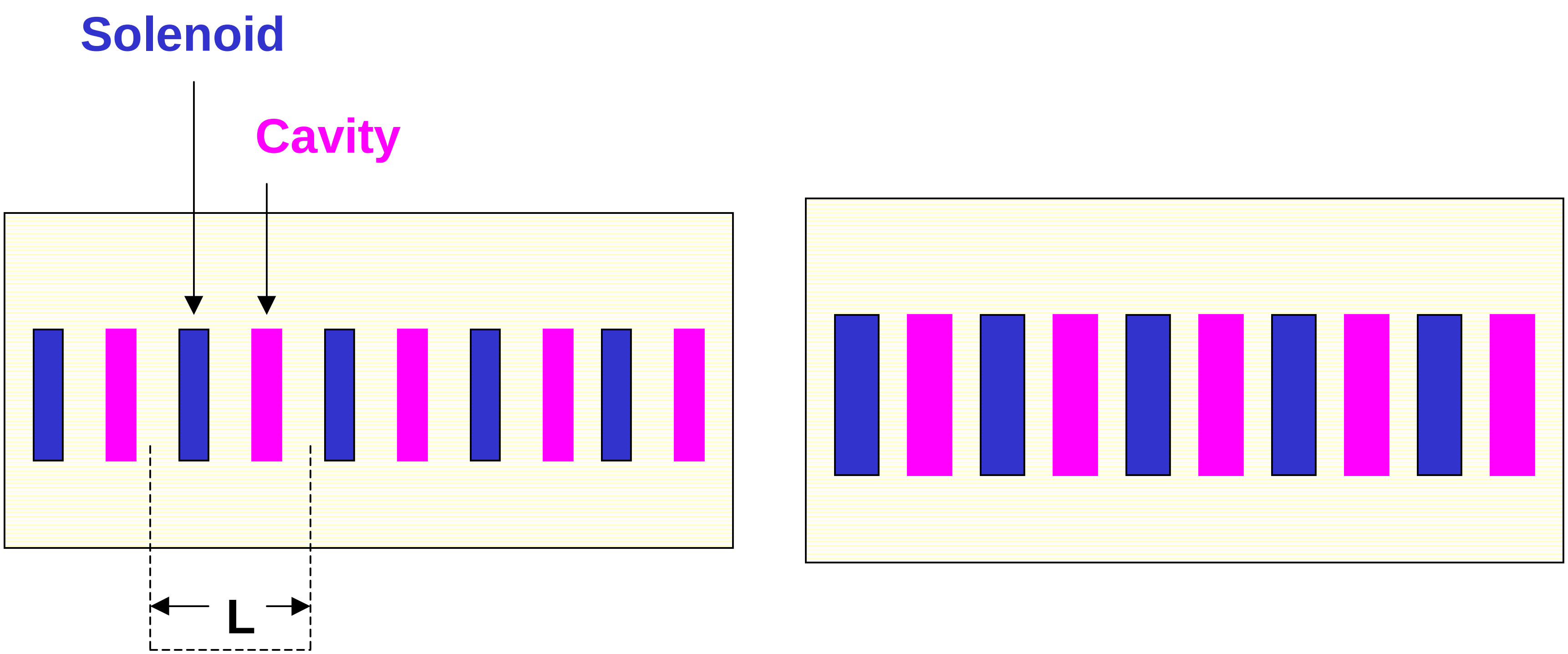
$$\langle E_0 T \rangle \le \frac{\pi mc^2 \gamma^3 \beta^3 \lambda}{8q \sin(-\phi) L_{Period}^2}.$$

- **The envelope instability constraint is important for high charge-to-mass ratio, low velocities***(cubic dependence)***, high frequencies, and long focusing periods** *(quadratic dependence).*
- **Reducing magnitude of phase ϕ below 30 deg doesn't help because phase width of bucket shrinks causing beam losses.**

A beam-dynamics approach for compact low-velocity proton superconducting linacs

- **Each cryomodule has identical elements and is a short FODO lattice with its characteristic period L.**
- **Allow period to change from one cryomodule to the next.**
  -Do not require that focusing period must be large enough to span the large space between cryomodules.
- **Shorten the focusing period.**
  -Include only one cavity and one solenoid per focusing period.
  -For compactness use solenoids instead of quadrupole multiplets for transverse focusing.
- **Use cavities and solenoids at both ends of cryomodule for matching between cryomodules.**
- **Gradients are still limited by $\sigma_0 < 90°$ requirement but these measures help.**

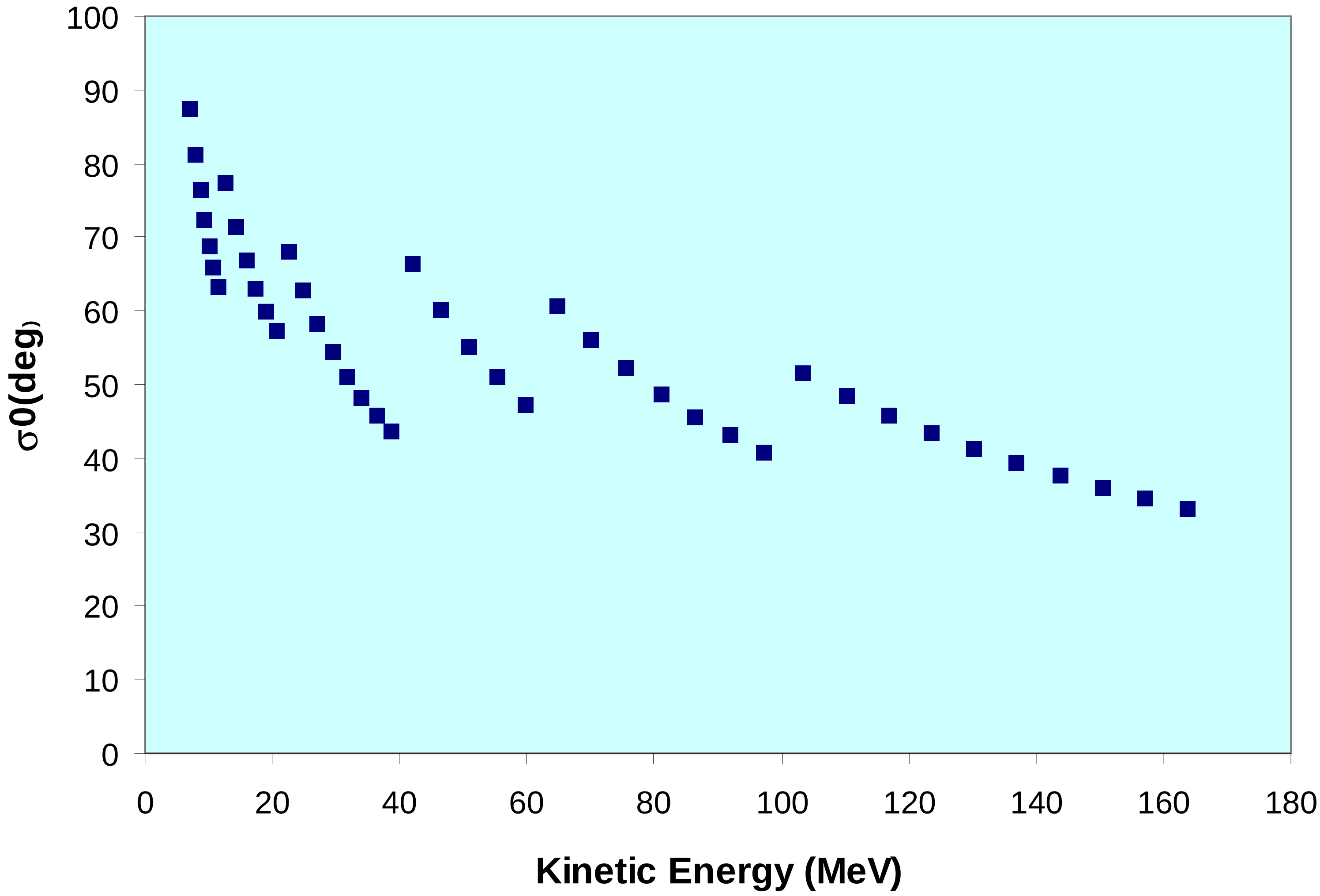
Example of two cryomodules: Cryomodules are short FODO lattices with different focusing periods. Each period consists of one cavity and one solenoid.

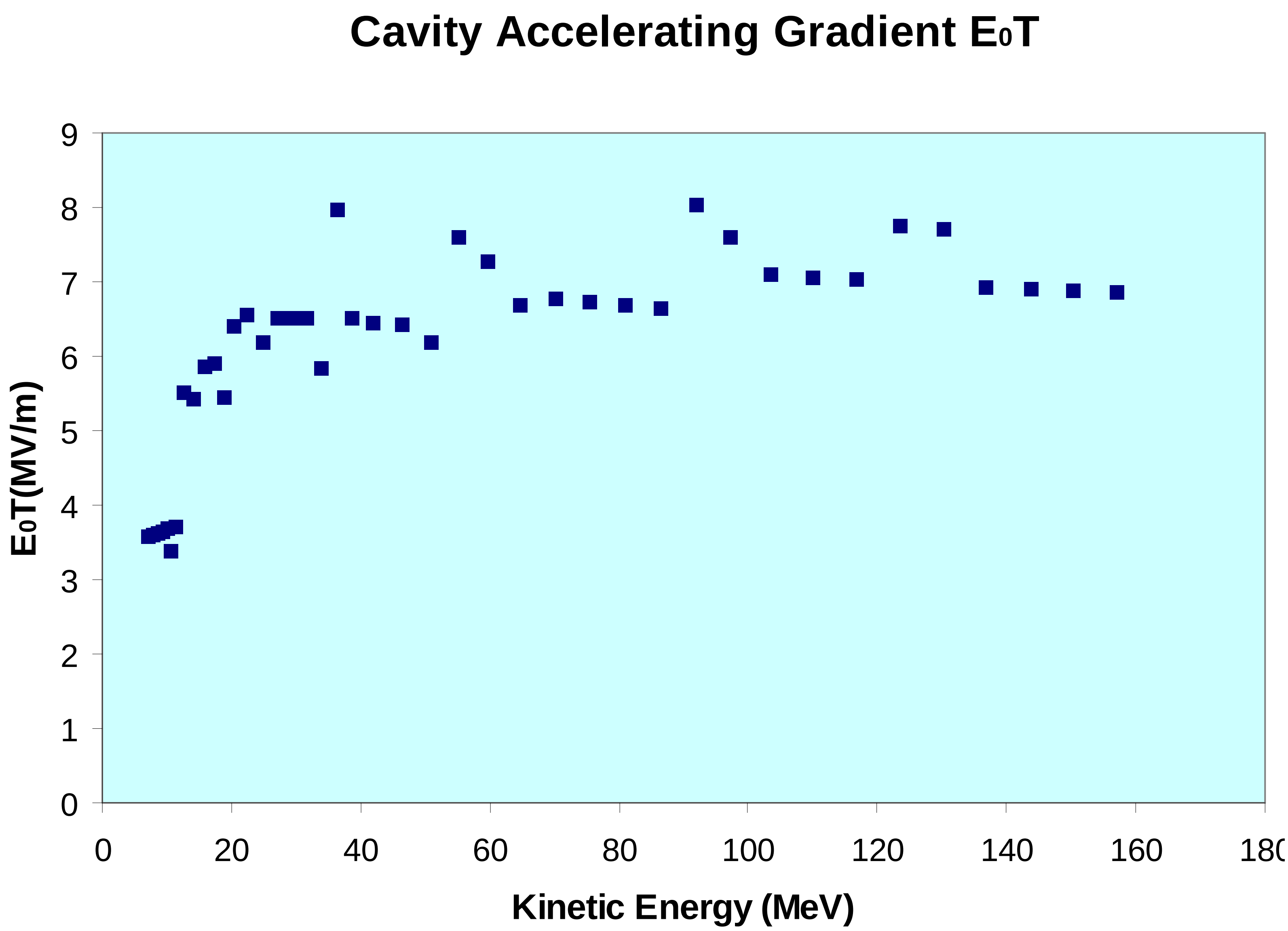


Cryomodule (β_1)

Cryomodule 2 (β_2)

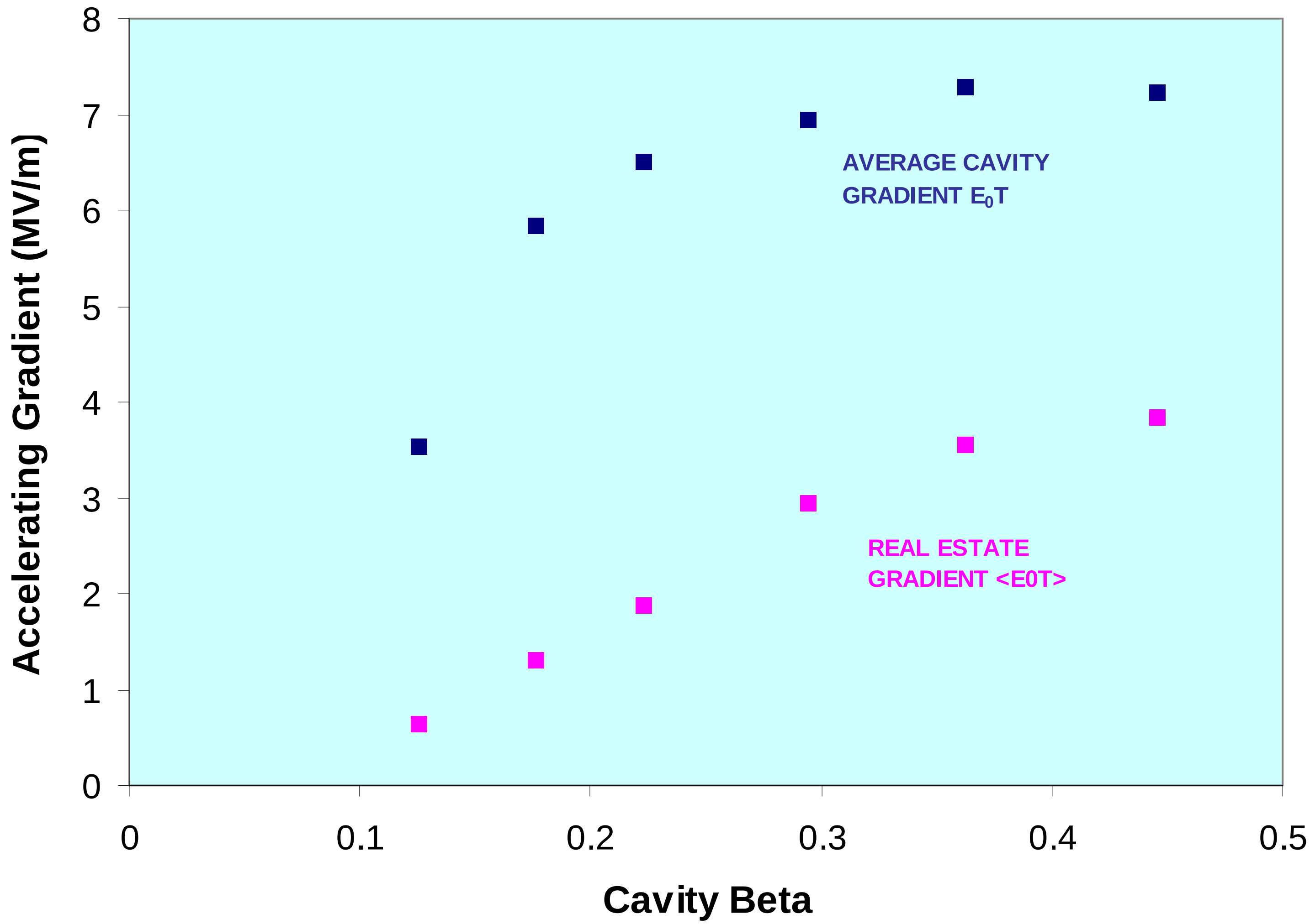
Longitudinal Zero Current Phase Advance Per Period



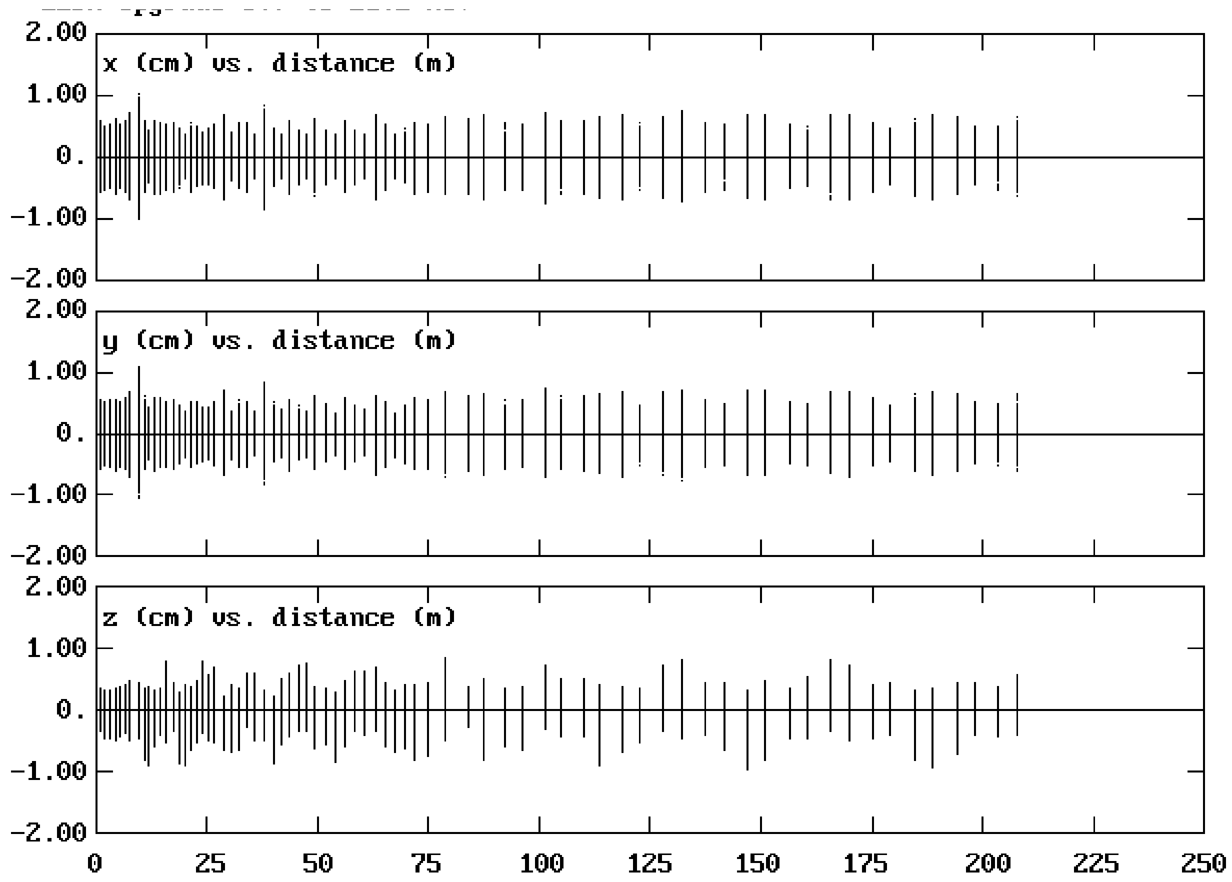


Cavity Accelerating Gradient E_0T
E_0T(MV/m)
Kinetic Energy (MeV)
0
1
2
3
4
5
6
7
8
9
0
20
40
60
80
100
120
140
160

Average Accelerating Gradients for Constant-β Linac Sections



Beam profiles for 8 superconducting sections from 6.7 to 600 MeV after approximate matching between cryomodules. Matching not perfect but satisfactory.



Conclusions

- **The longitudinal envelope instability can limit the accelerating gradient at low-velocities. It is more of a problem for proton (higher frequency and higher q/m) rather than heavy ion superconducting linacs.**

- Our longitudinal beam-dynamics design approach has been to keep $\sigma_0 < 90°$ and **minimize the focusing period**.

- The cryomodules form piecewise constant FODO lattices where each period contains one cavity and one solenoid.

- For 350-MHz proton linac in β range of **0.2 to 0.5 (20 to 150 MeV)** we could use cavity gradients up to about **8 MV/m** without longitudinal beam-dynamics problems.